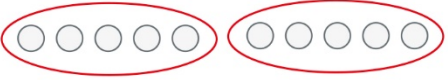
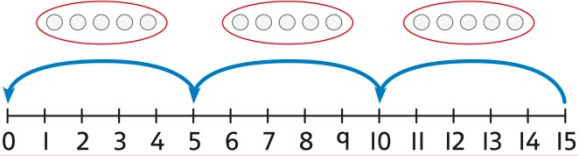
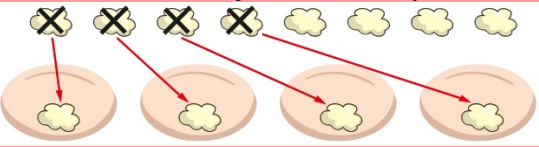
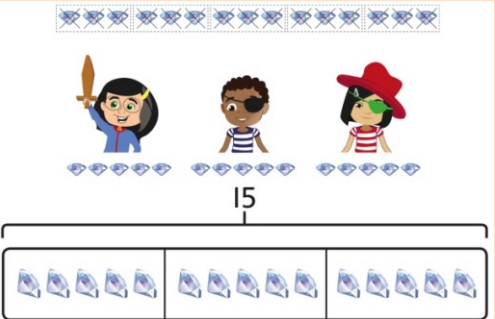
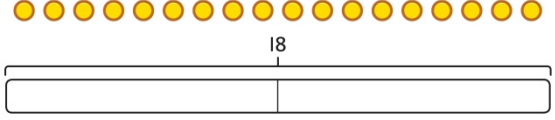
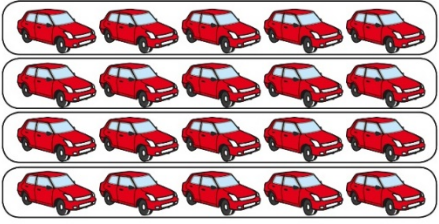
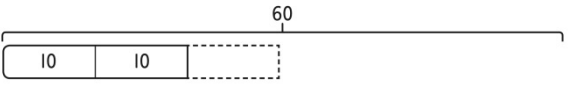


Year 1

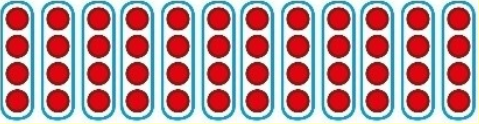
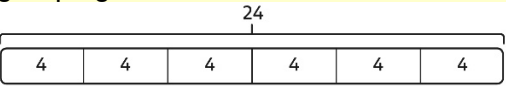
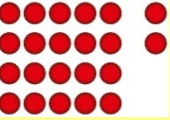
	Concrete	Pictorial	Abstract
Grouping	Learn to make equal groups from a whole and find how many equal groups of a certain size can be made. Sort a whole set people and objects into equal groups.  <i>There are 10 children altogether. There are 2 in each group. There are 5 groups.</i>	Represent a whole and work out how many equal groups.  <i>There are 10 in total. There are 5 in each group. There are 2 groups.</i>	Children may relate this to counting back in steps of 2, 5 or 10. 
Sharing	Share a set of objects into equal parts and work out how many are in each part. 	Sketch or draw to represent sharing into equal parts. This may be related to fractions. 	

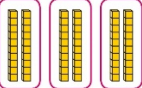
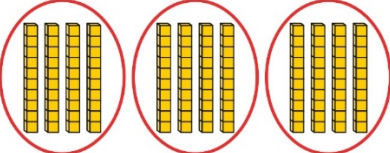
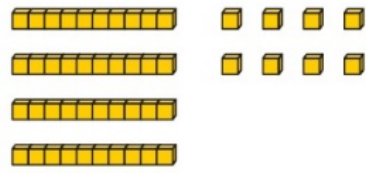
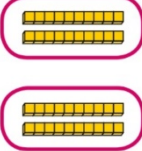
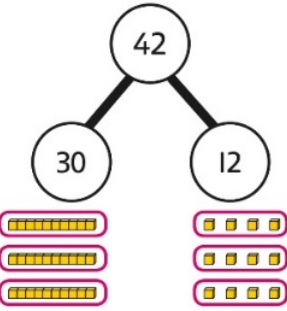
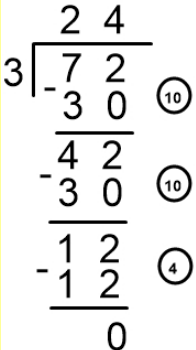
Year 2

	Concrete	Pictorial	Abstract
Sharing equally	Start with a whole and share into equal parts, one at a time. Start to understand how this also relates to grouping. To share equally between 3 people, take a group of 3 and give 1 to each person. Keep going until all the objects have been shared  <i>They get 5  each. 15 shared equally between 3 groups. They get 5 each.</i>	Represent the objects shared into equal parts using a bar model.  <i>20 shared into 5 equal parts. There are 4 in each part.</i>	Use a bar model to support understanding of the division.  $18 \div 2 = 9$

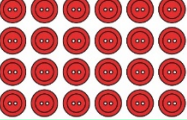
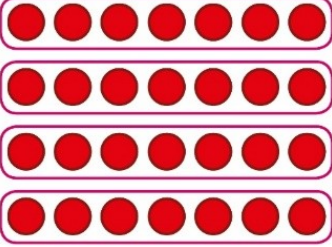
Grouping equally	Understand how to make equal groups from a whole.  8 divided into groups of 2. There are 4 equal groups.	Understand the relationship between grouping and the division statements. $12 \div 3 = 4$  12 divided into groups of 3. There are 4 equal groups.	Count in groups to relate division to repeated addition. $12 \div 3 = ?$ I will count in groups of 3. 3...6...9...12 There are 4 groups of 3.
Using known times-tables to solve divisions	Understand the relationship between multiplication facts and division.  There are 5 cars in each group. 4 groups of 5 cars is 20 cars in total. 20 divided by 4 is 5.	Use a bar model to support counting in groups to link known times-table facts.  $60 \div 10 = ?$ How many groups of 10 are in 60? I will count in 10s.	Relate times-table knowledge directly to division. I know that 3 groups of 10 makes 30, so I know that 30 divided by 10 is 3. $3 \times 10 = 30 \quad \text{so} \quad 30 \div 10 = 3$

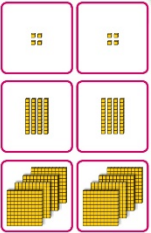
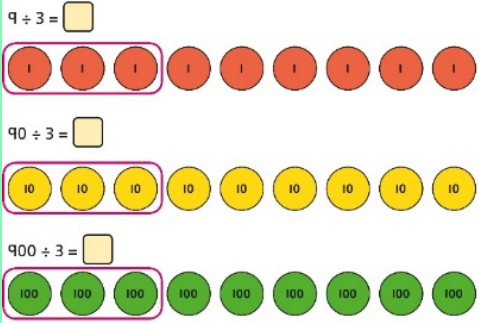
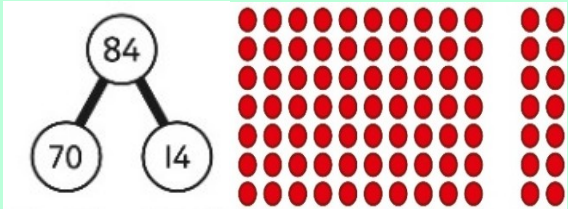
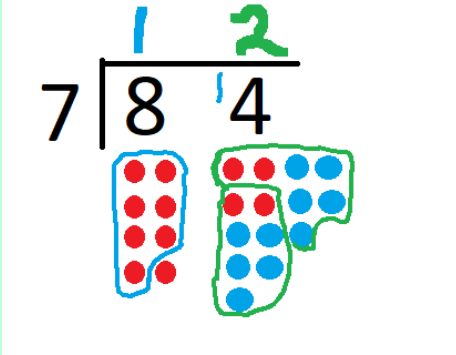
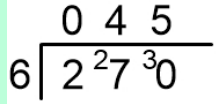
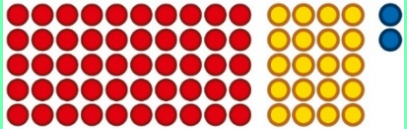
Year 3

	Concrete	Pictorial	Abstract
Using times-tables knowledge to divide	Use knowledge of known times-tables to calculate divisions by grouping  48 divided into groups of 4. There are 12 groups. $4 \times 12 = 48$ $48 \div 4 = 12$	Use knowledge of known times-tables and bar models to calculate divisions by grouping.  I know that $6 \times 4 = 24$ so I know that $24 \div 4 = 6$.	Use knowledge of known times-tables and repeated addition to calculate divisions by grouping. I need to work out $24 \div 4$. I can count in group of 4. 4...8...12...16...20...24 There are 6 groups of 4 in 24.
Understanding remainders	Use equipment to understand that a remainder occurs when a set of objects cannot be divided equally any further.  There are 13 sticks in total. There are 3 groups of 4, with 1 remainder.	Use images to explain remainders.  $22 \div 5 = 4 \text{ remainder } 2$	Understand that the remainder is what cannot be shared equally from a set. $22 \div 5 = ?$ $4 \times 5 = 20$ $5 \times 5 = 25 \dots \text{this is larger than } 22$ I can make 4 equal groups with 2 left over. So, $22 \div 5 = 4 \text{ remainder } 2$
Using known facts to divide multiples of 10	Use place value equipment to understand how to divide by unitising.	Divide multiples of 10 by unitising.	Divide multiples of 10 by a single digit using known times-tables.

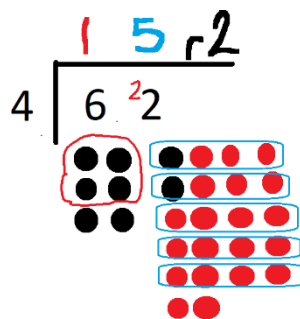
	Make 6 ones divided by 3.  Now make 6 tens divided by 3.  What is the same? What is different?	 12 tens shared into 3 equal groups. 4 tens in each group.	$180 \div 3 = ?$ 180 is 18 tens. 18 divided by 3 is 6. 18 tens divided by 3 is 6 tens. $18 \div 3 = 6$ $180 \div 3 = 60$
2-digit number divided by 1-digit number.	 $48 \div 2 = ?$ First divide the 10s.  Then divide the 1s. 	$42 \div 3 = ?$ I need to find 10x the divisor to help me partition.  $30 \div 3 = 10$ $12 \div 3 = 4$ $10 + 4 = 14$ $42 \div 3 = 14$	Children divide by repeated subtraction (chunking).  Find 10 x the divisor and subtract. Write the groups at the side of the calculation. Continue until you can no longer take away a chunk of 10 groups. Count in groups of the divisor. Write how many groups you can make at the side of the calculation. When you can make no more groups, add the number of groups and write as the quotient.

Year 4

	Concrete	Pictorial	Abstract
Understanding the relationship between multiplication and division, including times-tables	Use objects to explore families of multiplication and division facts.  $4 \times 6 = 24$ 24 is 6 groups of 4. 24 is 4 groups of 6.	Represent divisions using an array.  $28 \div 7 = 4$	Understand families of related multiplication and division facts. I know that $5 \times 7 = 35$ so I know all these facts: $5 \times 7 = 35$ $7 \times 5 = 35$ $35 = 5 \times 7$ $35 = 7 \times 5$ $35 \div 5 = 7$ $35 \div 7 = 5$

	24 divided by 6 is 4. 24 divided by 4 is 6.		$7 = 35 \div 5$ $5 = 35 \div 7$
Dividing multiples of 10 and 100 by a single digit	Use place value equipment to understand how to use unitising to divide.  8 ones divided into 2 equal groups = 4 ones in each group 8 tens divided into 2 equal groups = 4 tens in each group 8 hundreds divided into 2 equal groups = 4 hundreds in each group	Represent divisions using place value equipment.  $9 \div 3 =$ $90 \div 3 =$ $900 \div 3 =$ $9 \div 3 = 3$ 9 tens $\div 3 =$ 3 tens. 9 hundreds $\div 3 =$ 3 hundreds.	Use known facts to divide 10s and 100s by a single digit. $15 \div 3 = 5$ $150 \div 3 = 50$ $1500 \div 3 = 500$
Dividing 2-digit and 3-digit numbers by a single digit.	Use place value equipment to explore why different partitions are needed. Represent how to partition flexibly on a part-whole model. $84 \div 7 = ?$  I will partition into 70 and 14 because I am dividing by 7. $70 \div 7 = 10$ $14 \div 7 = 2$ $10 + 2 = 12$.	Use place value equipment alongside short division. Work with divisions that require exchange. 	More confident mathematicians may begin to see the link between grouping counters and counting in groups of. 
Understanding remainders	Represent the remainder as the part that cannot be grouped equally. 	Calculate as short division and record the remainder as part of the quotient.	More confident mathematicians may begin to see the link between grouping counters and counting in groups of.

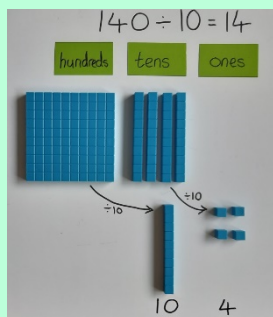
$$72 \div 5 = 14 \text{ remainder } 2$$



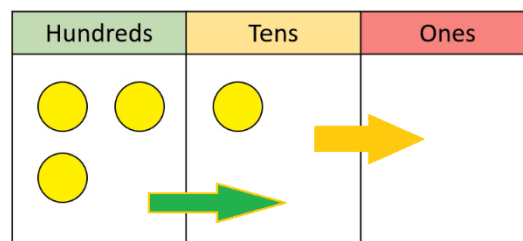
$$4 \overline{) 23 \text{ r } 3}$$

Divide a number by 10 and 100.

Understand division by 10 using exchange.

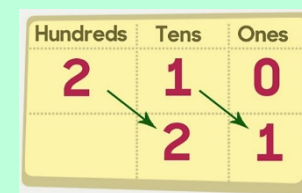


Use a place value chart to model:



$$310 \div 10 = 31$$

Understand the movement of digits on a place value grid.



Year 5

Understanding factors and prime numbers

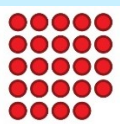
Use equipment to explore the factors of a given number.



$$24 \div 3 = 8$$

$$24 \div 8 = 3$$

8 and 3 are factors of 24 because they divide into 24 exactly.



5 is not a factor of 24 because there is a remainder.

Pictorial

Understand that prime numbers are numbers with exactly two factors.

$$13 \div 1 = 13$$

$$13 \div 2 = 6 \text{ r } 1$$

$$13 \div 4 = 4 \text{ r } 1$$

1 and 13 are the only factors of 13.
13 is a prime number.

Abstract

Understand how to recognise prime and composite numbers.

- I know that 31 is a prime number because it can be divided by only 1 and itself without leaving a remainder.

- I know that 33 is not a prime number as it can be divided by 1, 3, 11 and 33.

- I know that 1 is not a prime number, as it has only 1 factor.

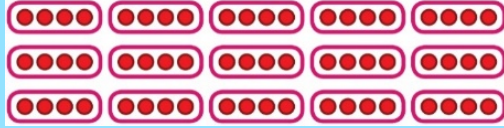
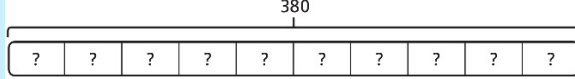
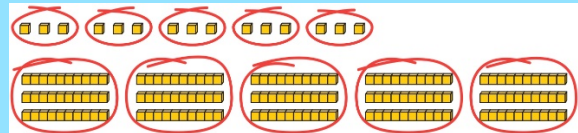
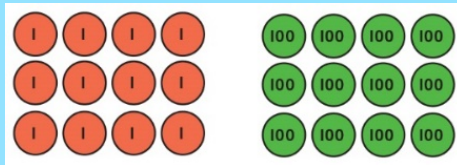
Understanding inverse operations and the link with multiplication,

Use equipment to group and share and to explore the calculations that are present.

I have 28 counters.

Represent multiplicative relationships and explore the families of division facts.

Represent the different multiplication and division relationships to solve missing number problems requiring inverse operations.

grouping and sharing	- I can make 7 groups of 4. - I can share them equally into 7 groups. There are 4 in each group. -I can make groups of 4. There are 7 equal groups.	 $60 \div 4 = 15$ $60 \div 15 = 4$	$22 \div ? = 2$ $22 \div 2 = ?$ $? \times 2 = 22$ $2 \times ? = 22$										
Dividing whole numbers by 10, 100 and 1,000		Use a bar model to support dividing by unitising. $380 \div 10 = 38$  380 is 38 tens. $38 \times 10 = 380$ $10 \times 38 = 380$ So, $380 \div 10 = 38$	Understand how and why the digits change on a place value grid when dividing by 10, 100 or 1,000. <table border="1" data-bbox="1590 410 2013 494"><tr><td>Th</td><td>H</td><td>T</td><td>O</td></tr><tr><td>3</td><td>2</td><td>0</td><td>0</td></tr></table> $3,200 \div 100 = ?$ $3,200$ is 3 thousands and 2 hundreds. $200 \div 100 = 2$ $3,000 \div 100 = 30$ $3,200 \div 100 = 32$ <i>So, the digits will move two places to the right.</i>	Th	H	T	O	3	2	0	0		
Th	H	T	O										
3	2	0	0										
Dividing by multiples of 10, 100 and 1,000	Use place value equipment to represent known facts and unitising.  15 ones put into groups of 3 ones. There are 5 groups. $15 \div 3 = 5$ 15 tens put into groups of 3 tens. There are 5 groups. $150 \div 30 = 5$	Represent related facts with place value equipment when dividing by unitising.  12 ones divided into groups of 4. There are 3 groups. 12 hundreds divided into groups of 4 hundreds. There are 3 groups. $1200 \div 400 = 3$	Reason from known facts, based on understanding of unitising. Use knowledge of the inverse relationship to check. $3,000 \div 5 = 600$ $3,000 \div 50 = 60$ $3,000 \div 500 = 6$ $5 \times 600 = 3,000$ $50 \times 60 = 3,000$ $500 \times 6 = 3,000$										
Dividing up to four digits by a single digit using short division	Explore grouping using place value equipment. $268 \div 2 = ?$ <i>There is 1 group of 2 hundreds.</i>	Use place value equipment on a place value grid alongside short division. Work with divisions that require exchange.	Use short division for up to 4-digit numbers divided by a single digit. <table border="1" data-bbox="1590 1445 1859 1546"><tr><td></td><td>0</td><td>5</td><td>5</td><td>6</td></tr><tr><td>7</td><td>3</td><td>8</td><td>9</td><td>2</td></tr></table>		0	5	5	6	7	3	8	9	2
	0	5	5	6									
7	3	8	9	2									

There are 3 groups of 2 tens.
There are 4 groups of 2 ones.

$264 \div 2 = 134$

$4 \overline{) 9 \ 2}$

T	O

First, lay out the problem.

$4 \overline{) 9 \ 1 \ 2}$

T	O

How many groups of 4 go into 9 tens?
2 groups of 4 tens with 1 ten left over.

$4 \overline{) 2 \ 1 \ 2}$

T	O

Exchange the 1 ten left over for 10 ones.
We now have 12 ones.

$4 \overline{) 2 \ 3 \ 1 \ 2}$

T	O

How many groups of 4 go into 12 ones?
3 groups of 4 ones.

$3,892 \div 7 = 556$

Use multiplication to check.

$6 \times 7 = 42$
 $50 \times 7 = 350$
 $500 \times 7 = 3500$

$3,500 + 350 + 42 = 3,892$

Understanding remainders

Understand remainders using concrete versions of a problem.

80 cakes divided into trays of 6.

80 cakes in total. They make 13 groups of 6, with 2 remaining.

Use short division and understand remainders as the last remaining 1s.

$6 \overline{) 8 \ 0}$

T	O

$6 \overline{) 1 \ 3 \ 2 \ 0}$

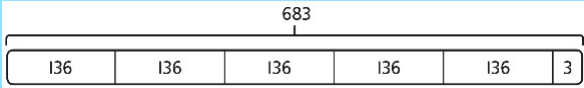
T	O

$6 \overline{) 1 \ 3 \ 2 \ 0} \text{ r } 2$

T	O

Lay out the problem as short division.
There is one group of 6 tens. There are 2 tens remaining.
There are 3 groups of 6 ones. There are 2 ones remaining.

In problem solving contexts, represent divisions including remainders with a bar model.



$683 = 136 \times 5 + 3$
 $683 \div 5 = 136 \text{ r } 3$

Dividing decimals by 10, 100 and 1,000

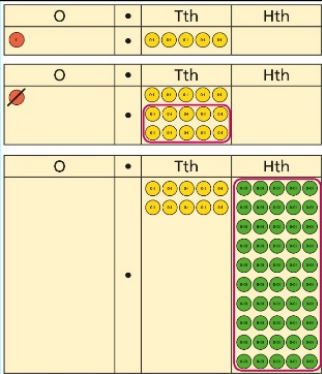
Understand division by 10 using exchange.

2 ones are 20 tenths.

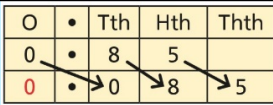
Represent division using exchange on a place value grid.

Understand the movement of digits on a place value grid.

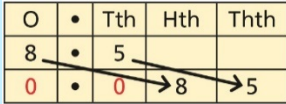
20 tenths divided by 10 is 2 tenths.



1.5 is 1 one and 5 tenths.
This is equivalent to 10 tenths and 50 hundredths.
 $10 \text{ tenths} \div 10$ is 1 tenth.
 $50 \text{ hundredths} \div 10$ is 5 hundredths.
 $1.5 \div 10$ is 1 tenth and 5 hundredths.
 $1.5 \div 10 = 0.15$



$$0.85 \div 10 = 0.085$$

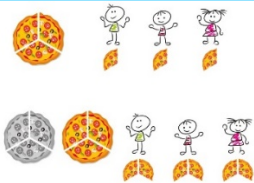


$$8.5 \div 100 = 0.085$$

Understanding the relationship between fractions and division

Use sharing to explore the link between fractions and division.

1 whole shared between 3 people.
Each person receives one-third.



Use a bar model and other fraction representations to show the link between fractions and division.



$$1 \div 3 = \frac{1}{3}$$

Use the link between division and fractions to calculate divisions.

$$5 \div 4 = \frac{5}{4} = 1 \frac{1}{4}$$

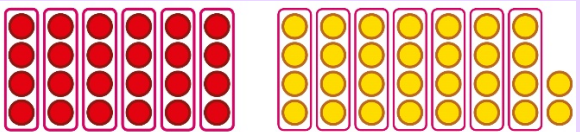
$$11 \div 4 = \frac{11}{4} = 2 \frac{3}{4}$$

Year 6

Understanding factors and primes

Concrete

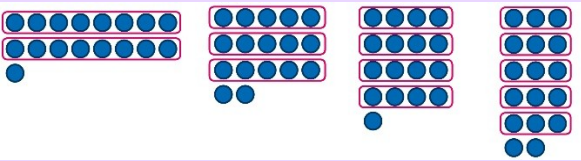
Use equipment to explore different factors of a number.



$24 \div 4 = 6$ $30 \div 4 = 7 \text{ remainder } 2$
4 is a factor of 24 but is not a factor of 30.

Pictorial

Recognise prime numbers as numbers having exactly two factors. Understand the link with division and remainders.



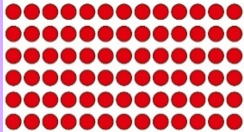
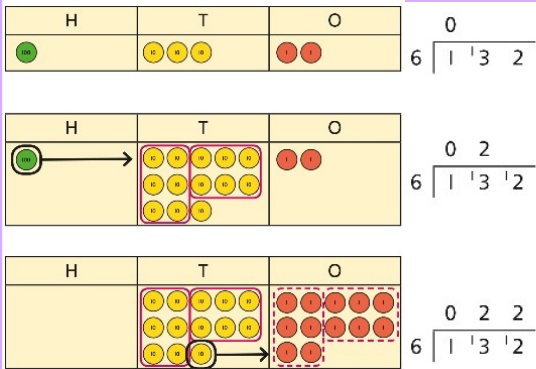
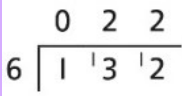
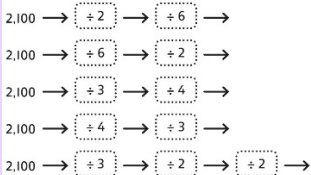
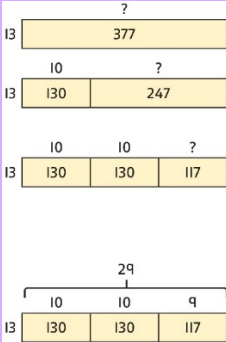
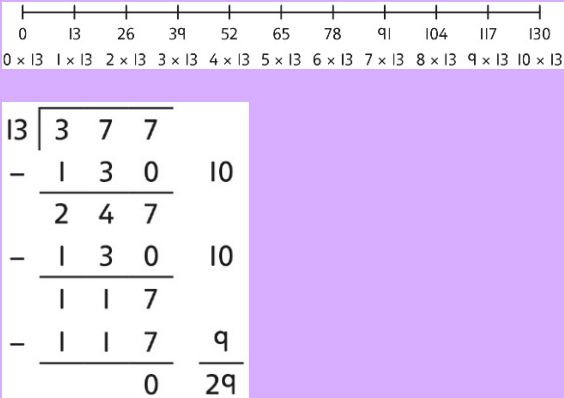
$$17 \div 2 = 8 \text{ r } 1$$
$$17 \div 3 = 5 \text{ r } 2$$
$$17 \div 4 = 4 \text{ r } 1$$
$$17 \div 5 = 3 \text{ r } 2$$

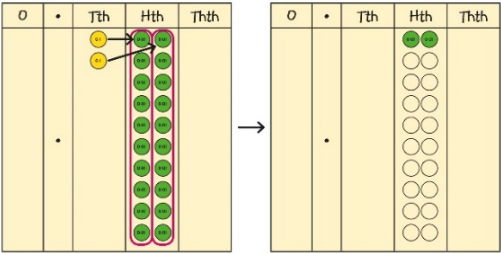
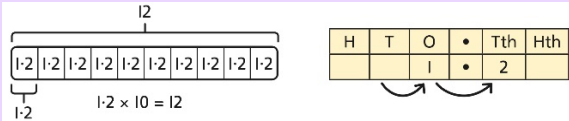
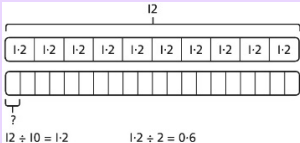
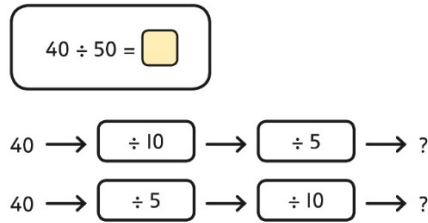
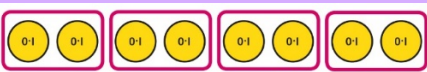
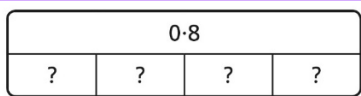
SPIDER

Abstract

Recognise and know primes up to 100. Understand that 2 is the only even prime, and that 1 is not a prime number.



Dividing by a single digit	Use equipment to make groups from a total.  There are 78 in total. There are 6 groups of 13. There are 13 groups of 6.	Use equipment to represent short division to divide by a single digit. 	Use short division to divide by a single digit. 
Dividing by a 2-digit number using factors	Understand that division by factors can be used when dividing by a number that is not prime.	Use factors and repeated division. $1,260 \div 14 = ?$  $1,260 \div 2 = 630$ $630 \div 7 = 90$ $1,260 \div 14 = 90$	Use factors and repeated division where appropriate. $2,100 \div 12 = ?$ 
Dividing by a 2-digit number using long division	Use equipment to build numbers from groups.  182 divided into groups of 13. There are 14 groups.	Use an area model alongside written division to model the process. $377 \div 13 = ?$  $377 \div 13 = 29$	Use long division where factors are not useful (for example, when dividing by a 2-digit prime number). Write the required multiples to support the division process. $377 \div 13 = ? \text{ SHORT DIVISION}$  $377 \div 13 = 29$

			A slightly different layout may be used, with the division completed above rather than at the side. $\begin{array}{r} 3 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \end{array}$ $\begin{array}{r} 38 \\ 21 \overline{) 798} \\ - 630 \\ \hline 168 \\ - 168 \\ \hline 0 \end{array}$ Divisions with a remainder explored in problem-solving contexts.
Dividing by 10, 100 and 1,000	Use place value equipment to explore division as exchange.  Exchange each 0.1 for ten 0.01s. Divide 20 counters by 10. <i>0.2 is 2 tenths. 2 tenths is equivalent to 20 hundredths. 20 hundredths divided by 10 is 2 hundredths.</i>	Represent division to show the relationship with multiplication. Understand the effect of dividing by 10, 100 and 1,000 on the digits on a place value grid.  Understand how to divide using division by 10, 100 and 1,000. $12 \div 20 = ?$  $12 \div 10 = 1.2$ $1.2 \div 2 = 0.6$	Use knowledge of factors to divide by multiples of 10, 100 and 1,000.  $40 \div 50 = 0.8$ $40 \rightarrow \div 10 \rightarrow \div 5 \rightarrow ?$ $40 \rightarrow \div 5 \rightarrow \div 10 \rightarrow ?$ $40 \div 5 = 8$ $8 \div 10 = 0.8$ So, $40 \div 50 = 0.8$
Dividing decimals	Use place value equipment to explore division of decimals.  <i>8 tenths divided into 4 groups. 2 tenths in each group.</i>	Use a bar model to represent divisions.  $4 \times 2 = 8$ $8 \div 4 = 2$ So, $4 \times 0.2 = 0.8$ $0.8 \div 4 = 0.2$	Use short division to divide decimals with up to 2 decimal places.

			$\begin{array}{r} \cdot \\ 8 \overline{) 4 \cdot 2 \ 4} \\ 0 \cdot \\ 8 \overline{) 4 \cdot ^4 2 \ 4} \\ 0 \cdot 5 \\ 8 \overline{) 4 \cdot ^4 2 \ ^2 4} \\ 0 \cdot 5 \ 3 \\ 8 \overline{) 4 \cdot ^4 2 \ ^2 4} \end{array}$	
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